Prediction of ingress rates to turbine and compressor wheelspaces

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The paper uses the inviscid form of the momentum equation for rotating flows to analyse conditions in the space between a rotating and stationary disc—the "wheelspace." Its principal object is to predict the ingress from the external atmosphere when the imposed rate of radial flow to the wheelspace is less than required to suppress inflow. The analysis developed can deal with a varying external pressure along the seal, a situation now known to have a profound effect upon the amount of ingress. The inviscid assumption inevitably requires an empirical input, and critical parameters in determining sealing performance appear as the local ratio of the tangential fluid velocity in the wheelspace to that of the rotating disc and the seal inflow and outflow discharge coefficients. Although the former of these appears to be strongly dependent upon rate of imposed flow, rational values for these critical coefficients yield results in good agreement with experimental observation from test rigs simulating turbomachine stages. There is a need to build a database of such empirical coefficients for the use of system designers from a large amount of performance data from many designs of seal.

Keywords: turbine; rim seal; axial seal; radial seal; wheelspace; ingress; egress; sealing efficiency; rotating flow; rotor–stator; disc; asymmetric flow; pressure perturbation; discharge coefficients

Introduction

This paper uses the one-dimensional (1-D) inviscid momentum equation to model the flow in the gap between a rotating and stationary disc in a turbine or compressor, a space or volume known as the "wheelspace" and illustrated in Figure 1. Conditions in wheelspaces often are critical to the integrity of a turbomachine because the rotating members are highly stressed. Therefore, it is a clear requirement for designers accurately to determine the temperatures to which they are subject. These temperatures are set largely by the extent to which there is ingress of working fluid from the external mainstream to the wheelspaces, and this is controlled by a peripheral sealing system through which there is inevitably a radially outward flow of fluid.

In the present paper, the integrated form of the equation of motion is used to predict the difference in pressure between the external environment, the working annulus of a turbomachine, and that in the wheelspace at the radius where the peripheral seal between rotating and stationary members begins to influence the internal flow. This pressure difference is determined by the rate of any radial flow imposed outwardly from near the axis of the system. When there is no imposed radial flow, there is still an outflow through the seal driven by the pumping action of the rotating disc, and there must be a corresponding ingress of fluid from the external atmosphere drawn into the system through part of the seal to satisfy continuity, and there is a negative pressure with respect to the annulus in the wheelspace. If the rate of imposed radial flow is sufficiently large, the wheelspace pressure becomes essentially positive, and ingress from the external flow can theoretically be prevented with only outflow from the wheelspace occuring. The conditions required to achieve this ideal situation have been widely studied, early on by Bayley and Owen (1970), more recently, among many others by Chew et al. (1994) and reviewed by Owen and Rogers (1989).

The principal object of the present paper is to predict the rate of ingress to a wheelspace and the corresponding efficiency of the peripheral seal, defined as (1 − ingress/total flow through wheelspace), when the imposed radial flow falls below that required to prevent ingress and ensure only outflow in the seal gap. The bounding conditions, corresponding at one extreme to no imposed flow and at the other, to just zero ingress, are used to evaluate the empirical coefficients controlling the system. The analytical procedure proves to be applicable to the condition in which the pressures in the external environment vary peripherally along the seal, a situation which has been shown by much recent work to influence critically the rate of wheelspace ingress in ways well predicted by the present analysis.

The inviscid equation of motion

In an early paper on shrouded rotor–stator systems (Bayley and Owen 1970), the equation of motion for an assumed inviscid core

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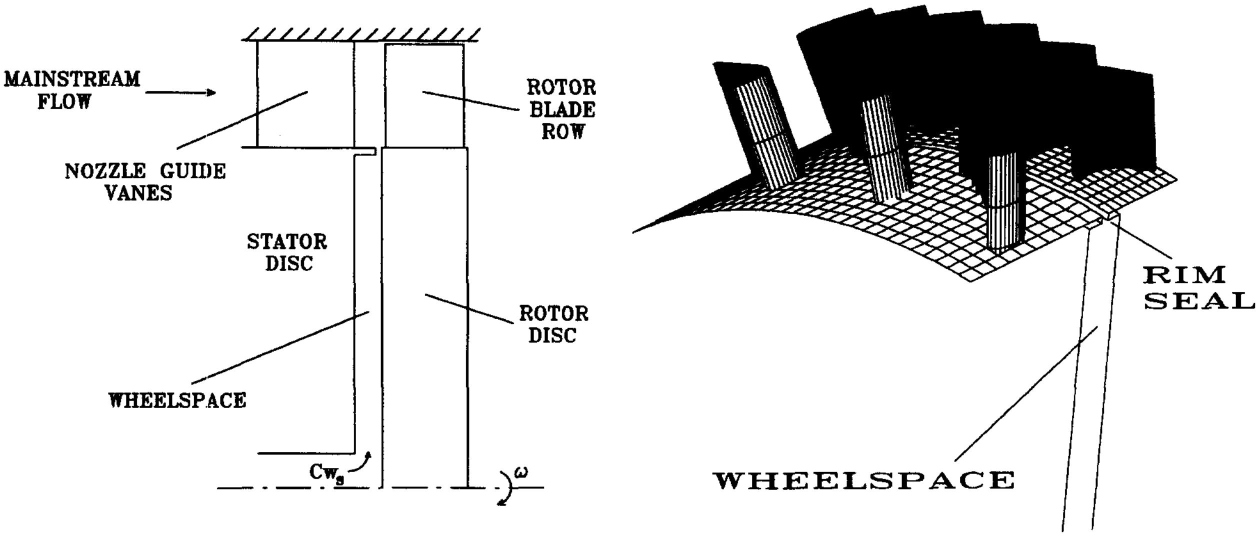

Figure 1 Schematic representation of the rotor–stator wheelspace geometry

of fluid

$$-\frac{1}{\rho}\frac{dp}{dr} = V_r\frac{dV_r}{dr} - \frac{V_\phi^2}{r} \qquad (1)$$

is integrated between radii r and r_0 to the dimensionless form

$$\Delta Cp = ACw^2 - B\mathrm{Re}_\phi^2 \qquad (2)$$

in which A and B are taken as constants. As is shown later, these contain the geometric parameters for the experimental apparatus and, more particularly for the present exercise, assumed constants appertaining to the flows in the system. One of these, k_2, represents the fraction of the local disc surface tangential velocity at which the inviscid core is assumed to rotate, and to which we return later. The other flow coefficient k_1 defines the mean radial velocity for the main wheelspace gap.

Bayley and Owen (1970) argued that setting the left-hand side of Equation 2; that is, the difference in static pressure between the wheelspace near to the outer radius and the external environment, to zero corresponded to the condition at which the superimposed radial flow Cw_s in dimensionless form, was just sufficient to prevent mainstream ingress. This leads to the convenient and commonly used equation for this flow, $Cw_{\min}$:

$$Cw_{\min} = \text{Constant} \times G_c \mathrm{Re}_\phi \qquad (3)$$

The constant in Equation 3 is set in the analysis by the geometry of the system and the values used for the empirical coefficients

Notation

A, B	constants in equation of motion
c	seal clearance
Cw	dimensionless rate of radial flow, $m/\mu\ r_0$
Cw_{in}	dimensionless ingress flow
Cw_{out}	dimensionless egress flow
$Cw_{\min}$	dimensionless rate of imposed radial flow just to prevent ingress
Cw_s	dimensionless rate of imposed radial flow
Cw_{total}	dimensionless total radial outflow
Cp	dimensionless wheelspace pressure, $p\rho r_0^2/\mu^2$
G	dimensionless wheelspace gap, s/r_0
G_c	dimensionless seal clearance, c/r_0
k_1	radial continuity coefficient, defined by $m = k_1 2\pi r s \rho V_r$
k_2	core rotation fraction, defined as $V_\phi/\omega r$
kc	local coefficient of discharge
kc_{in}	coefficient of discharge for inflow
kc_{in1}	coefficient of discharge for inflow for zero superposed flow
kc_{out}	coefficient of discharge for outflow
N_{blades}	number of pressure perturbations or nozzle guide vanes
$N_{\theta\mathrm{increments}}$	number of circumferential increments
p	wheelspace pressure
p_a	external, ambient pressure
Dp_a	half amplitude of external pressure variation
r	radius
r_1	radius at which seal geometry begins to effect wheelspace flow
r_0	outer radius
Re_ϕ	rotational Reynolds number, $\rho\omega r_0^2/\mu$
s	gap between rotor and stator
s'	axial gap at radius r
V_r	radial component of fluid velocity
V_ϕ	tangential component of fluid velocity
Greek	
θ	circumferential location
μ	viscosity
ρ	density
ω	angular velocity of rotating disc

in equation 1. In Bayley and Owen (1970), which used data from experiments on a wheelspace shrouded by a simple axial seal in a quiescent atmosphere, the constant was proposed as 0.61, a value which has been extensively and apparently reliably used for conditions consistent with those of the original experiments. Much later work has, however, demonstrated that the efficacy of real sealing systems is, of course, dependent upon their detailed geometry and upon the conditions in the environment external to the wheelspace—the mainstream annulus flow in a turbine or compressor—especially the asymmetry of the external flow.

A re-analysis (Bayley 1993) of the experimental observations of radial pressure distribution in the original 1970 paper suggests that the pressures are consistent with k_1, the imposed rate of radial flow/$(2\pi\rho r s V_r)$ having a value in the wheelspace of about unity, greater than originally proposed largely through an inconsistency in its definition compared with the usual concept of a coefficient of discharge. The value, unity, equates to the rational procedure of using the average radial velocity at radius r in integrating the tangential momentum equation to determine the distribution of static pressure in the wheelspace and is now firmly recommended.

The second coefficient k_2, which relates the average local tangential component of fluid velocity V_ϕ to that of the rotating disk ωr, is found, as might be anticipated, to be heavily dependent upon the totality of prevailing conditions. It can range from as low as 0.2 or less in a quiescent symmetrical atmosphere, to values well above 0.5 for conditions in a turbine and is known to be dependent upon the rate of any superimposed radial flow as well as the rotational speed itself (Owen and Rogers 1989). It emerges as an important parameter to the system designer who has to predict with confidence the value of Cw_{min}, the rate of superimposed flow required to prevent all ingress of external environment to the wheelspace—theoretically and idealistically, at least. Of at least equal importance to the designer, especially given the uncertainties associated with Cw_{min} in a complex environment, is the prediction of rates of ingress as the rate of radial flow falls below Cw_{min}, because these are crucial in determining disk temperatures.

Inflow with variable superimposed radial flow

The situation to be examined is that in which the rate of superimposed radial flow is varied down to zero from values above Cw_{min} predicted from Equations 2 or 3, when the wheelspace pressure near to the periphery becomes equal to that in the surrounding atmosphere. Integration of Equation 1 gives:

$$-\frac{p}{\rho} = (k_2\omega)^2\frac{r^2}{2} - \frac{V_r^2}{2} + \text{constant} \tag{4}$$

This equation is applied across the rotating system from the radius, say r_1, at which the outward flow comes under the influence of the contraction to the seal, to the outer extremity of the rotor at r_0, where the pressure becomes that of the environment p_a, so that

$$-\frac{p - p_a}{\rho} = (k_2\omega)^2\left(\frac{r_1^2}{2} + \frac{r_0^2}{2}\right) - \left(\frac{V_{rl}^2}{2} - \frac{V_{r0}^2}{2}\right) \tag{5}$$

in which, as has been seen, $V_r = m/(kc2\pi\rho r s')$ with r, the appropriate radius, s' the gap or axial dimension at that radius, and kc a local "coefficient of discharge" corresponding to the local rate of radial flow m, and defined as k_1 in the main wheelspace gap.

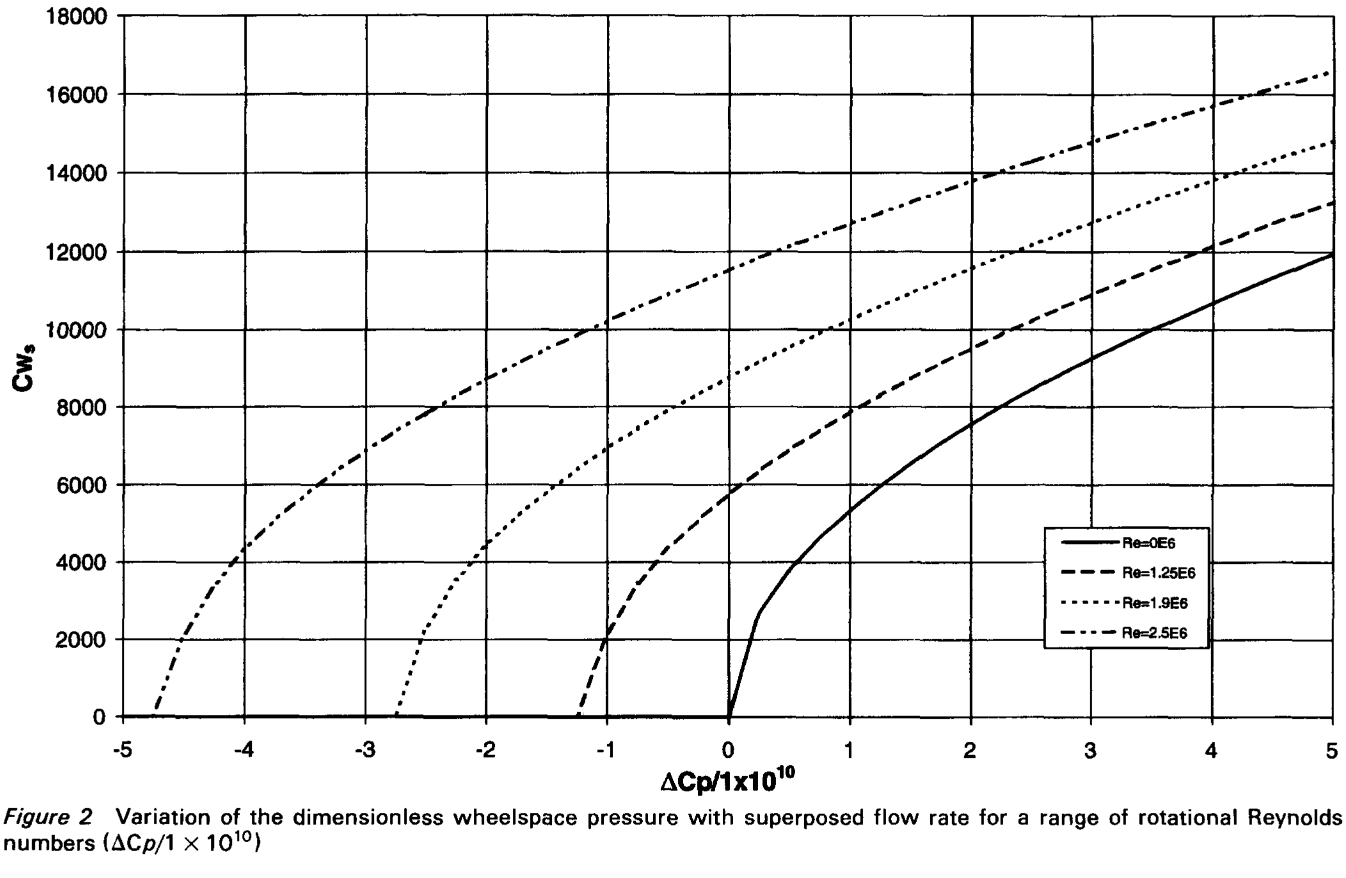

Figure 2 Variation of the dimensionless wheelspace pressure with superposed flow rate for a range of rotational Reynolds numbers ($\Delta Cp/1 \times 10^{10}$)

This equation can be written nondimensionally to define the parameters A and B in Equation 2 as

$$\Delta C_p = \frac{Cw^2}{8\pi^2}\left[\frac{1}{(kc_{out}G_c)^2} - \left(\frac{r_0/r_1}{k_1 G}\right)^2\right] - 0.5(k_2\mathrm{Re}_\phi)^2[1-(r_1/r_0)^2] \qquad (6)$$

Note that implicit in this form is the assumption that the tangential velocity fraction k_2 remain constant over the domain of integration.

The equation can be rearranged to give the rate of superimposed radial flow from the axis of a wheelspace required to develop a pressure difference represented dimensionlessly as ΔCp, between the radius r_1 and the outer radius, thus

$$Cw_s = \sqrt{[(\Delta Cp + B\mathrm{Re}_\phi^2)/A]} \qquad (7)$$

Equation 7 clearly shows the effect anticipated from the physics of the system—that the rate of superimposed radial flow Cw_s must rise to maintain a given pressure difference at the radius r_1 in the wheelspace as the rotational speed increases. Furthermore, as shown in Figure 2, at a given Reynolds number, as the specified pressure differences are reduced, the calculated value of Cw_s falls but remains positive, even for negative values of ΔCp, until this becomes $-B\mathrm{Re}_\phi$, which is the pressure deficit at r_1 due to rotation. With $Cw_s = 0$; i.e., no imposed radial flow, there must be inflow at the wheelspace periphery equal to the rate at which fluid is entrained by the rotating member of the system. Sambo (1983) showed, as might reasonably be anticipated from the physics of the system, the close relation between the rate of entrainment and Cw_{min}, with typically the latter, the imposed radial flow to prevent all ingress to the wheelspace from a quiescent, infinite atmosphere, about 20% greater than the entrainment rate. For the calculations in this paper, with the pressure at r_1 equal to $-B\mathrm{Re}_\phi^2$, the inflow with $Cw_s = 0$ has, as a starting assumption, been taken as equal to Cw_{min}.

It can be assumed further that, when the imposed radial flow Cw_s is less than Cw_{min}, the resulting ingress is driven by the simple Bernoulli equation in which the driving pressure difference is the pressure deficit at the radius r_1, ΔCp. The constant B in the form used in Equation 7 is a function of the imposed radial flow Cw_s, because k_2 is a function of this variable as well as of the rotational Reynolds number Re_ϕ and radius. A further critical parameter in determining the rate of inflow will be the "coefficient of discharge" representing the proportion of the area of the seal arrangement occupied by this inflow and for which the symbol kc_{in} is used. In this situation, the rate of inflow is simply represented from Bernoulli's equation in dimensionless form as

$$Cw_{in} = 2\pi kc_{in}G_c\sqrt{2\Delta Cp} \qquad (8)$$

For the present calculations, kc_{in} has been determined from the assumption described above, which is that when Cw_s is zero, and the pressure deficit is $B\mathrm{Re}_\phi^2$ the rate of inflow is Cw_{min}. From Equation 7 this is $\sqrt{B/A}\,\mathrm{Re}_\phi$, which is used in the computation as a starting value to iterate until Cw_{min} is determined as the value of Cw_s, at which ingress just becomes zero, and consistency between the coefficients is established. As reported in the Comparison with experimental observations section later in this paper, experimental observations show that the coefficient kc_{in} is heavily dependent upon the rate of imposed radial flow and appears to decrease exponentially as Cw_s increases from zero. The same value of the coefficient kc_{in} cannot be assumed to determine the rate of outflow for a given pressure excess, although when only this condition obtains, the value is of no immediate engineering significance because the sealing efficiency in which we are principally interested is then 100%. The outflow coefficient becomes more important, however, in controlling net flows in an asymmetrical external environment, which was noted earlier as being significant for rates of ingress and is considered in the next section. The outflow coefficient appears not to be strongly influenced by the rate of superimposed flow.

Figure 3 shows how the total rate of radial flow is predicted to vary by these procedures for a range of superimposed flows at rotational Reynolds numbers varied from 10^6 to 10^7. To derive these curves, the flow coefficient for the main wheelspace k_1 has been taken as unity, as proposed earlier in this paper. The fractional tangential velocity k_2 is taken from Bayley (1993) as 0.25 in the main wheelspace, but allowed to vary with Re_ϕ and Cw_s, as reported in Owen and Rogers (1989). The gap width G, is 0.1, and the seal clearance G_c is 0.01. At zero rate of imposed flow, Figure 3 shows the high rate of radial outflow corresponding to the entrainment rate of the rotating disc, equal by continuity to the rate of inflow to the wheelspace. As has been noted, this is taken to be Cw_{min}, determined as the rate of imposed flow at which ingress just becomes zero. As the rate of superimposed radial flow is first increased above zero, the rate of radial flow falls off rapidly, because the rate of inflow is heavily attenuated by the imposed flow according to the exponential relationship consistent with experimental observation. As the rate of imposed radial flow increases further so does the rate of outflow, the two becoming equal when all inflow is suppressed at $Cw_s = Cw_{min}$.

The rates of inflow plotted in Figure 3 can be used to determine a "sealing efficiency," defined as

$$1 - \frac{\text{ingress}}{\text{superimposed flow} + \text{ingress}} = \frac{\text{superimposed flow}}{\text{superimposed flow} + \text{ingress}} \qquad (9)$$

and such values are plotted as percentages in Figure 4. (A flowchart outlining the procedure for determining Cw_{min} is given in Appendix A.)

Pressure perturbations in the external environment

The data in the previous section are artificial, because they assume a quiescent constant property atmosphere beyond the wheelspace. Research on the wheelspace ingress problem subsequent to the early work in Bayley and Owen (1970), especially from operational machinery, showed that even small perturbations in the external atmosphere around a wheelspace could have a significant effect upon the sealing efficiency, a surprising observation given the parabolic nature of the equations determining the wheelspace flows even when written in the full form for viscous flow. The effects of the external flow were established as arising from static pressure perturbations or asymmetries in the external atmosphere which allowed local ingress (Phadke and Owen 1988), and these can be included conveniently in the present analysis.

The inevitable variations in pressure in the mainstream flow external to the wheelspace, usually represented in relation to the prevailing dynamic pressure, in modern high-speed machines can be significant indeed. In applying Equations 6 and 7 to estimate rates of net radial flow, the external pressure p_a can be allowed to vary along the extent of a sealing system. The procedure adopted takes no account of dynamic effects due to the pressure

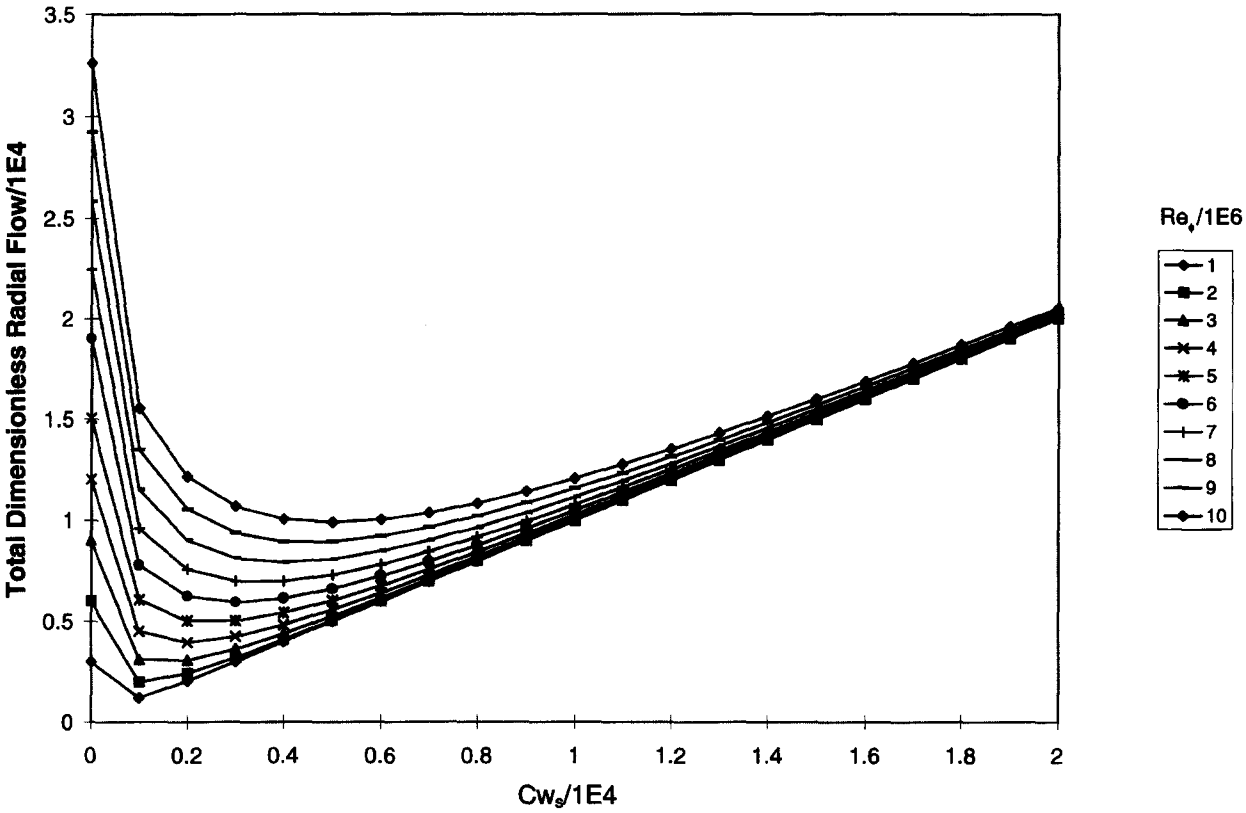

Figure 3 Variation of the total radial flow with superposed flow rate for a range of rotational Reynolds numbers ($Cw_s/1 \times 10^4$, $Cw_{total}/1 \times 10^4$)

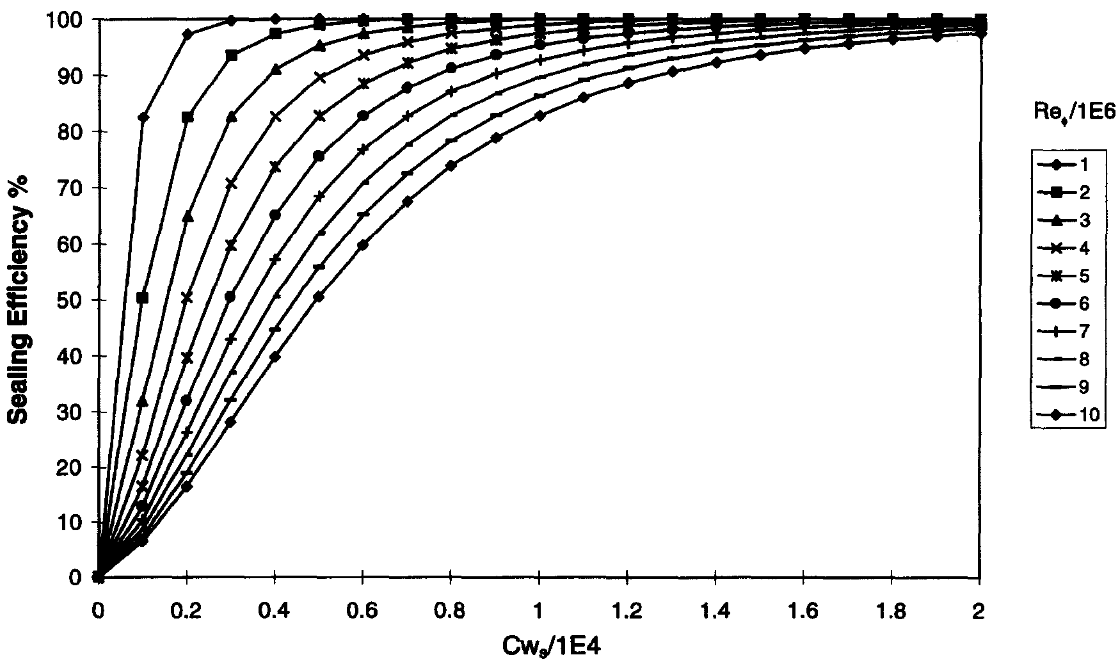

Figure 4 Variation of sealing efficiency with superposed flow rate for a range of rotational Reynolds numbers ($Cw_s/1 \times 10^4$)

perturbations and is, to that extent, a pseudostatic analysis. For the calculations for Figure 5, the static pressure p_a was varied by $\pm 10\%$ of the mainstream absolute pressure with ten complete cycles of sinusoidal variation over what, in practice, would be the full peripheral extent of a seal. Such a variation, which is not untypical of those in modern machines, will affect the seal performance if the coefficients of discharge through the seal clearance for inflow and outflow conditions are different, as discussed above for the condition when the imposed flow is insufficient to balance locally the boundary-layer entrainment rate. The discharge coefficients in the gap were taken again as for Figures 3 and 4 and as described above for inflow from the pressure condition when Cw_s is zero.

Compared with the earlier figure for an axisymmetric flow, the sealing efficiencies can now be seen to be less dependent upon the Reynolds number of the flow, especially at the lower rates of imposed flow, which is consistent with observations of such effects in test rigs and machines. The precise form of variation of the sealing efficiency is, of course, dependent upon the assumed values for the inflow and outflow coefficients of discharge, and further examination of experimental data is necessary to create a database of appropriate values. The procedure for determining Cw_{min} used previously, of iterating until ingress just becomes zero, is no longer valid, because even with perturbations as low as 1% in the external environment, low rates of inflow can continue even at high values of Cw_s. The values of Cw_{min} for unperturbed flow with the same geometry were used for the present calculations to evaluate the flow entrained by the rotating disc.

In a perturbed flow, even for modest levels of perturbation, because of the regions of low external pressure, total outflows greater than the sum of imposed flow and inflow can result. To satisfy continuity, this must be corrected by a reduction in wheelspace pressure to less than the Bernoulli pressure caused by the imposed flow alone. A subroutine is included in the program used in which the wheelspace pressure is reduced in successive iterations by amounts dependent upon the excess of the total outflow until this just balances the imposed flow plus the corrected rate of inflow. The procedure for determining ingress and egress flow rates is outlined by means of a flowchart in Appendix B.

The consequences of pressure variations in the atmosphere external to a wheelspace are naturally dependent upon the nature of these variations. The amplitude of variation is the critical parameter; for even small fluctuations, of one percent or so, have a significant effect, while large fluctuations can almost remove the effects of internal rotational Reynolds number upon Cw_{min}, and the tendency towards this result is shown by the bunching of the curves in Figure 5. The number or frequency of the perturbations appears to be much less important but could be significant with perturbations that are not themselves regular and symmetrical like the artificial sinusoids used in the present analysis.

Comparison with experimental observations

Green and Turner (1992) and Green (1993) report measurements of sealing efficiency in wheelspaces for a number of geometries, and Figures 6 and 7 compare their observations of sealing performance with the predictions of the above theory. Compared with the arbitrary values for the relevant constants used in the text above, values have been chosen for the figures to give the best agreement between observation and the prediction, shown as a full line and assuming a pressure perturbation in the external flow of $\pm 1\%$ of the mainstream absolute pressure. The selected empirical constants are given in Table 1. Green's experimental results were obtained using gas-sampling techniques at

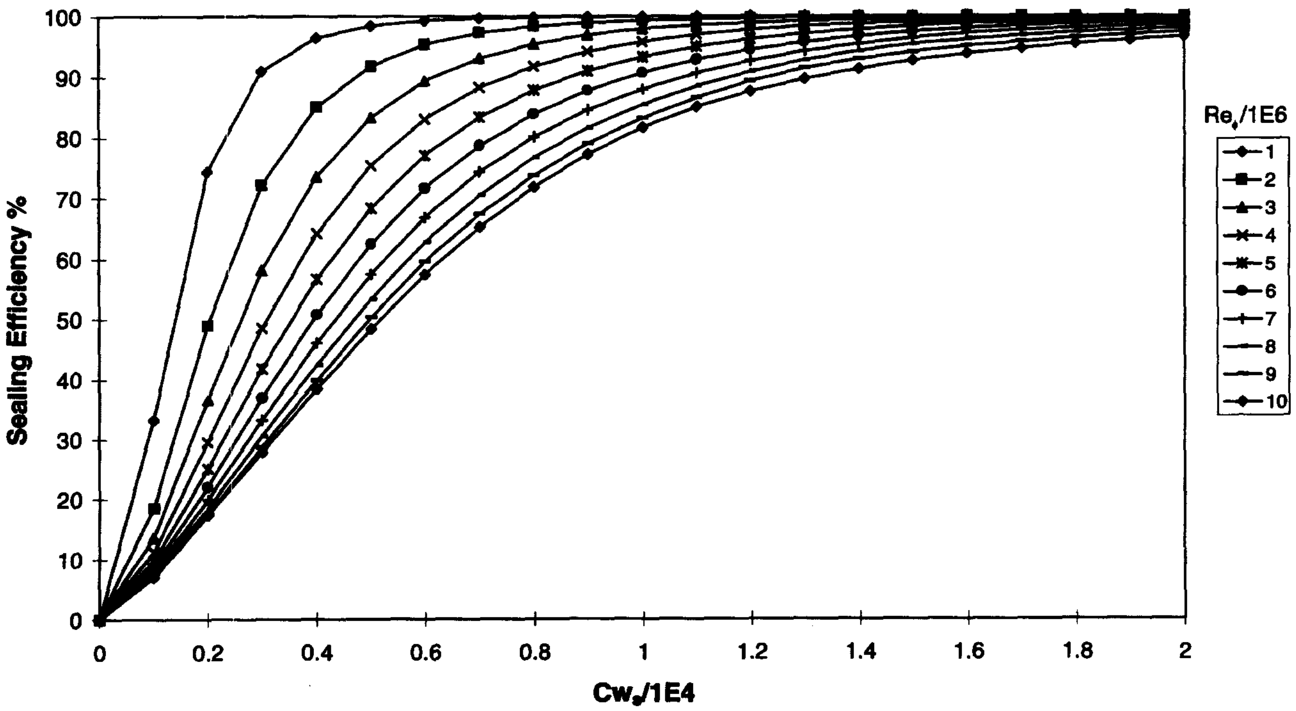

Figure 5 Variation of sealing efficiency with superposed flow rate for a range of rotational Reynolds numbers for the case of an asymmetric external pressure environment ($Cw_s/1 \times 10^4$)

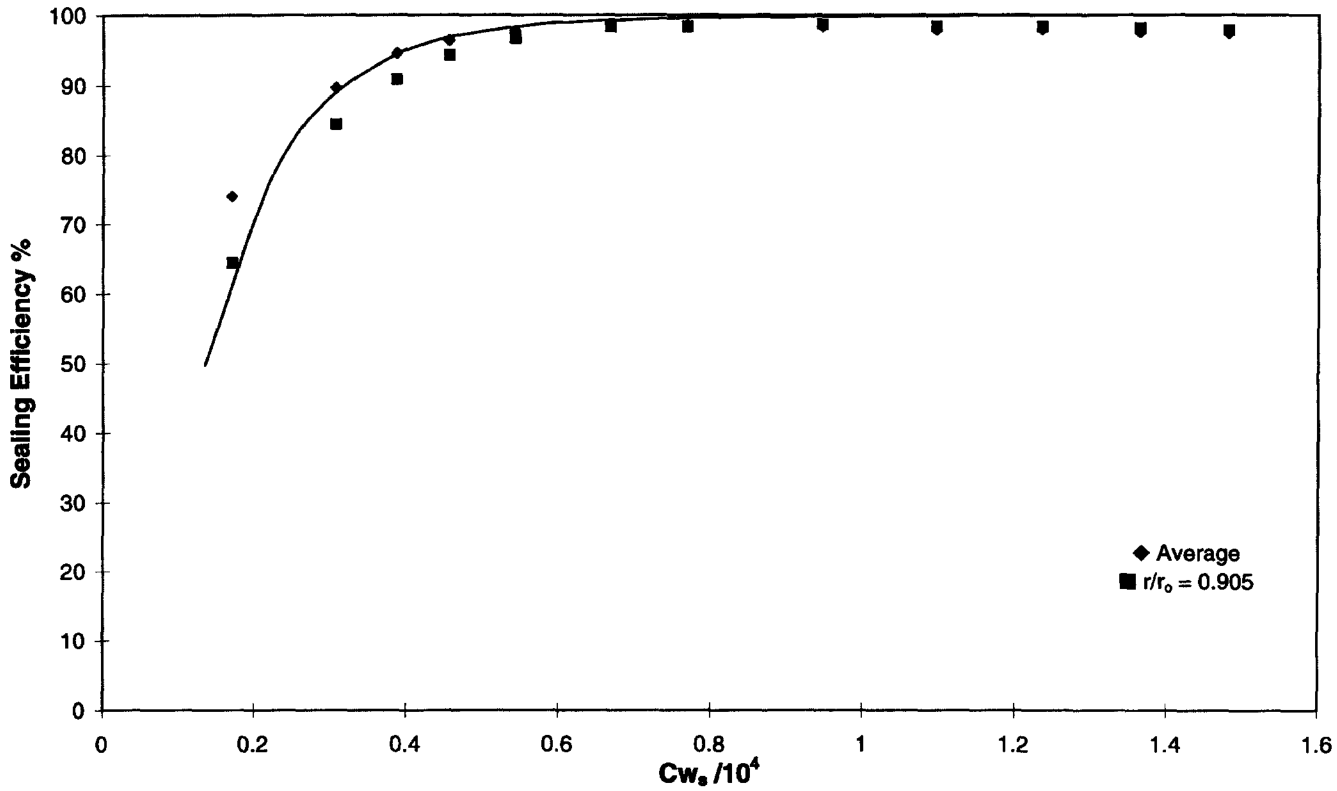

Figure 6 Comparison of the current model with the experimental observations of Green and Turner (1992) and Green (1993)—axial seal; the symbols represent experimental data, continuous lines represent predictions given by the model presented ($Cw_s/1 \times 10^4$)

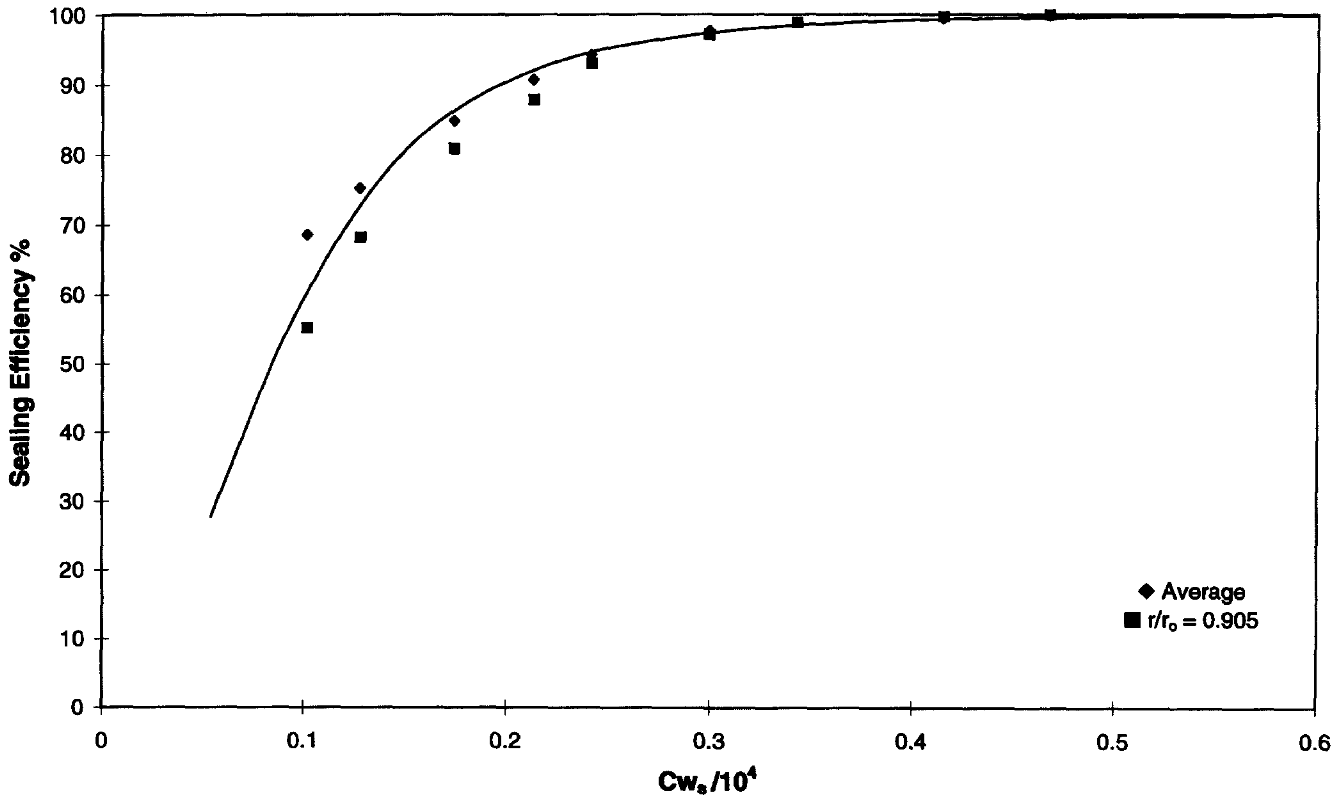

Figure 7 Comparison of the current model with the experimental observations of Green and Turner (1992) and Green (1993)—radial seal; the symbols represent experimental data, continuous lines represent predictions given by the model presented ($Cw_s/1 \times 10^4$)

Table 1 Empirical constants

Seal	G_c	k_1	k_2	kc_{out}	kc_{in}	$Cw_{min}/G_c Re_\phi$
Axial (Green)	0.01	1	$0.25/\mathrm{Exp}(5.7Cw_s/Re_\phi^{0.8}(r/r_0)^{2.6})$	0.8	$kc_{in1}/\mathrm{Exp}(5\sqrt{(Cw_s/Cw_{min})})$	0.38
Radial (Green)	0.01	1	$0.25/\mathrm{Exp}(5.7Cw_s/Re_\phi^{0.8}(r/r_0)^{2.6})$	0.55	$kc_{in1}/\mathrm{Exp}(5\sqrt{(Cw_s/Cw_{min})})$	0.23
Axial (Daniels)	0.0048	1	$0.75/\mathrm{Exp}(5.7Cw_s/Re_\phi^{0.8}(r/r_0)^{2.6})$	0.8	$kc_{in1}/\mathrm{Exp}(5\sqrt{(Cw_s/Cw_{min})})$	0.83
Axial* (Daniels)	0.0048	1	$0.85/\mathrm{Exp}(5.7Cw_s/Re_\phi^{0.8}(r/r_0)^{2.6})$	0.8	$kc_{in1}/\mathrm{Exp}(5\sqrt{(Cw_s/Cw_{min})})$	0.9
Radial (Daniels)	0.0024	1	$0.75/\mathrm{Exp}(5.7Cw_s/Re_\phi^{0.8}(r/r_0)^{2.6})$	0.55	$kc_{in1}/\mathrm{Exp}(5\sqrt{(Cw_s/Cw_{min})})$	0.8

*The external stream swirl for these tests was $2\omega r_0$; all others in Daniels et al. (1990) ωr_0

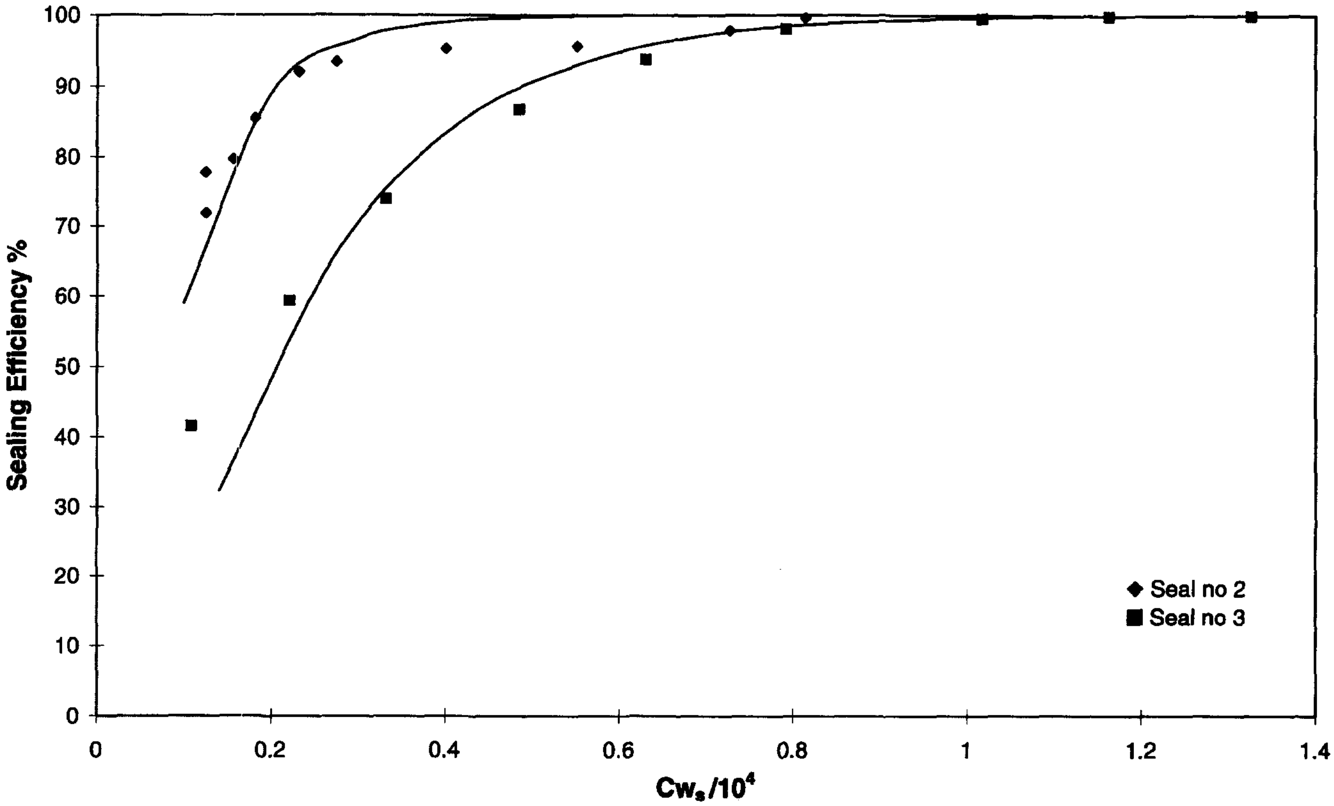

Figure 8 Comparison of the current model with the experimental observations of Daniels and Johnson et al. (1990)—radial seals; the symbols represent experimental data, continuous lines represent predictions given by the model presented ($Cw_s/1 \times 10^4$)

four radii, from $r/r_0 = 0.4$, where the sealing efficiency was consistently near 100%, to $r/r_0 = 0.905$ where the observations are as depicted in Figures 6 and 7.

The results in Figure 7 were obtained as average values with a radial seal, the associated theoretical predictions from which, as shown in Table 1, required only that kc_{out} be reduced to 0.55 compared with 0.8 for the earlier axial seal, a change entirely consistent with the more tortuous path to be taken by the flows in the radial seal and reflected also in the predicted value of the constant $Cw_{min}/G_c Re_\phi$. The latter is determined for the seal geometry by using the theory of the paper for an unperturbed external atmosphere and, as earlier described, using the analysis to predict the rate of superimposed flow at which the inflow was just zero.

Daniels and Johnson (1990) and their colleagues from Pratt & Whitney report data at higher Reynolds numbers from a pressurised rig which could provide a degree of swirl in the superimposed flow, and their results are shown in Figures 8 and 9. The values of the coefficients used to predict the results of these workers from axial and radial seals, again assuming a pressure perturbation of 1%, are shown also in Table 1 and, again, rationally represent the departure from Green's rig. The different values of $Cw_{min}/G_c Re_\phi$ between the rigs of Green (1993), Green and Turner (1992), and Daniels et al. (1990) are accounted for by the very different geometries used. The preswirl used in Daniels et al. explains the higher values of k_2.

Conclusion

In this paper, the 1-D inviscid equation of rotating flow has been used to predict flows and pressures in a wheelspace system. Estimates have been made of rates of inflow and outflow between the wheelspace and its environment, including the situation in which the latter is allowed to vary along the seal length. The amplitude of the variation is clearly critical and remains significant at low values, particularly in its role in attenuating the effect of rotational Reynolds number. The effect of the number or frequency of the pressure variations appears to be small. Inflow rates have been used to calculate sealing efficiencies, and

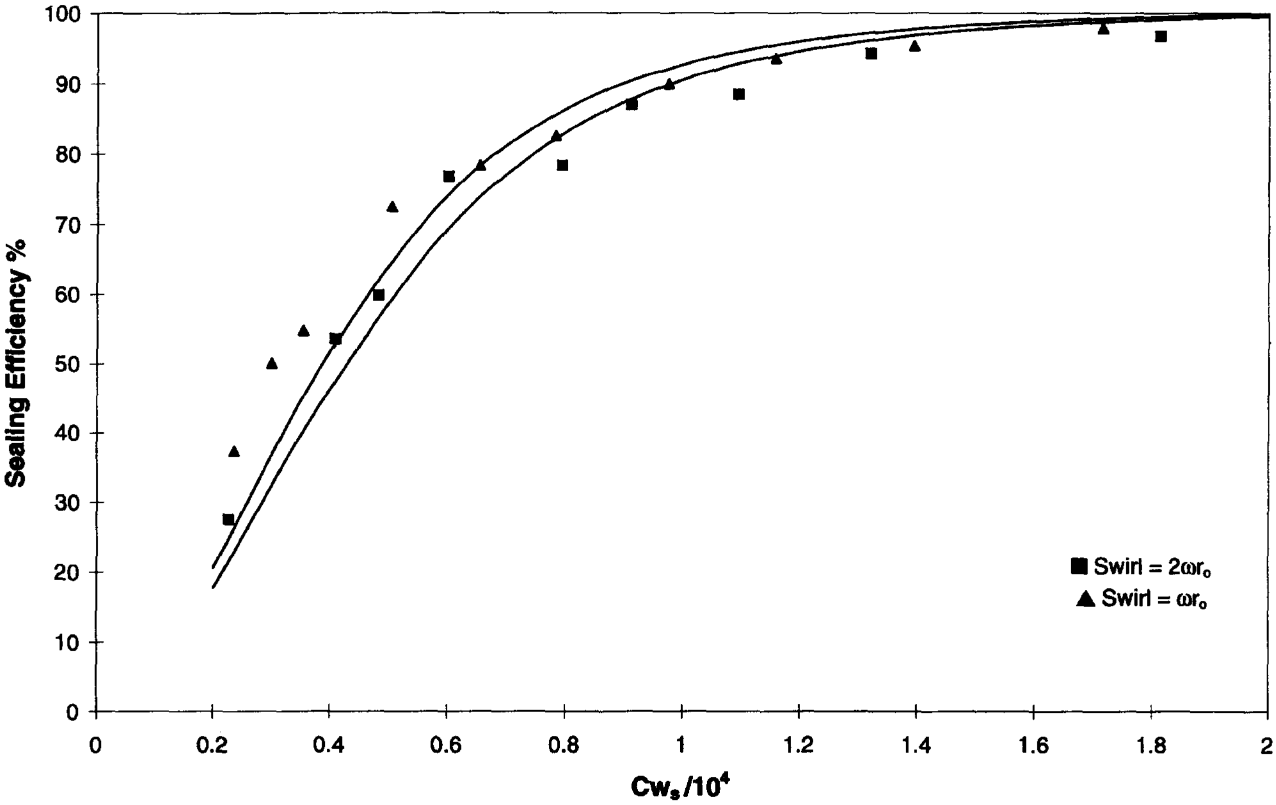

Figure 9 Comparison of the current model with the experimental observations of Daniels and Johnson et al. (1990)—axial seals; the symbols represent experimental data, continuous lines represent predictions given by the model presented ($Cw_s/1 \times 10^4$)

these are compared with experimental observations from different experimental rigs.

The analytical approach certainly simplifies the complex flow regimes and boundary conditions associated with the wheelspace system and, thus, inevitably requires a substantial empirical input. Comparison between prediction and observation indicates that the parameters which appear to be critical, are the tangential velocity fraction and the inflow and outflow coefficients of discharge. Generally, the changes in these coefficients rationally represent the physics of the experimental systems studied and suggest that the analysis, based on an inviscid model used originally and effectively to predict Cw_{min} (Bayley and Owen 1970), is a robust and realistic representation of the system. It is proposed that to create an extensive database for the use of machine designers, experimental data from a wide range of geometric conditions be analysed in terms of these coefficients, which would be expected together to reflect types of seals and their working environments.

Appendix A: model algorithms

Appendix A gives the procedure used to determine Cw_{min}, Cw_{in}, Cw_{out}.

Appendix B gives the procedure used for determining ingress and egress for given values of Re_ϕ and Cw_s for an asymmetric pressure environment.

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Appendix A.

Set Re_ϕ, kc_{out}, k_1, k_2

$$A = \frac{1}{8\pi^2}\left[\left(\frac{1}{kc_{out}G_c}\right)^2 - \left(\frac{1}{(r_1/r_0)Gk_1}\right)^2\right]$$

$$kc_{in1} = \frac{1}{2\pi G_c\sqrt{2A}}$$

Starting guess $Cw_{min} = \sqrt{B/A}\,\mathrm{Re}_\phi$

Loop until Cw_{min} converged for given Re_ϕ

Increment Cw_s

$$k_w = \frac{k_2}{e^{5.7Cw_s/\mathrm{Re}_\phi^{0.8}(r/r_0)^{2.6}}}$$

$$B = 0.5k_w^2[1-(r_1/r_0)^2]$$

$$kc_{in} = \frac{kc_{in1}}{e^{(\sqrt{Cw_s/0.04Cw_{min}})}}$$

If $(ACw_s^2 - B\,\mathrm{Re}_\phi^2) < 0$ then inflow $Cw_{in} = 2\pi kc_{in}G_c\sqrt{2(B\,\mathrm{Re}_\phi^2 - ACw_s^2)}$

If $(ACw_s^2 - B\,\mathrm{Re}_\phi^2) > 0$ then outflow $Cw_{out} = 2\pi kc_{out}G_c\sqrt{2(ACw_s^2 - B\,\mathrm{Re}_\phi^2)}$

If $Cw_{in} = 0$ set $Cw_{min} = Cw_s$

↓

Cw_{min}

Appendix B.

Set kc_{out}, k_1, k_2

$$A = \frac{1}{8\pi^2}\left[\left(\frac{1}{kc_{out}G_c}\right)^2 - \left(\frac{1}{(r_1/r_0)Gk_2}\right)^2\right]$$

$$kc_{in1} = \frac{1}{2\pi G_c\sqrt{2A}}$$

Set Re_ϕ, Cw_s

$$k_w = \frac{k_2}{e^{5.7Cw_s/Re_\phi^{0.8}(r/r_0)^{2.6}}}$$

$$B = 0.5k_w^2[1-(r_1/r_0)^2]$$

$$Cw_{min} = \text{constant} \times G_c\, Re_\phi$$

$$kc_{in} = \frac{kc_{in1}}{e^{(\sqrt{Cw/0.04Cw_{min}})}}$$

Loop until continuity satisfied

Increment θ

$$p_a = \frac{\rho r_0^2}{\mu^2}Dp_a\left[\sin\left(\frac{2\pi N_{blades}}{N_{\theta increments}}\right)\right]$$

If $(ACw_s^2 - B\,Re_\phi^2 - p_a) < 0$ then inflow $Cw_{in} = 2\pi kc_{in}G_c\sqrt{2(B\,Re_\phi^2 - ACw_s^2 + p_a)}$

If $(ACw_s^2 - B\,Re_\phi^2 - p_a) > 0$ then outflow $Cw_{out} = 2\pi kc_{out}G_c\sqrt{2(ACw_s^2 - B\,Re_\phi^2 - p_a)}$

Sum Cw_{in} and Cw_{out}

Check for continuity

Factor wheelspace pressure to correct for continuity if necessary

Cw_{in}, Cw_{out}